Modeling Basic Aspects of Cyber-Physical Systems, Part II


* Rice University, USA
**Halmstad University, Sweden
Motivation

- Educating designers
- Developing expressive, efficient, and robust modeling and simulation tool
Vision

Design better simulation tool to support virtual testing and analysis of cyber-physical systems at both industrial and academic levels.
Today to model and analyze a cyber physical system, one need to connect several domains, this brings forward the challenge of adding features to support innovative design process.
Key Idea

We are working on a tool called **Acumen**. Small language for hybrid systems modeling. We use Acumen to model and analyze two sophisticated standard **robotics** examples and add features to support **design innovation process**.
Newtonian and Lagrangian Formulations

• Why Newtonian?
  – Non-conservative forces
  – Vector base
  – Solve constraints equations

• Why Lagrangian?
  – Direct specification
  – Calculate potential and kinetic energies
  – Compact without constraints
Case Study 1- Quadcopter

- Complex mechatronic system
- 6-degree of freedom
- Underactuation
- Cover different domains
Case Study 1- Quadcopter

- Mathematical Model
  - Newton-Euler formulation
  - Vectors

\[
\sum F = m\ddot{a} = G + RT
\]

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\dot{\omega}_3 \\
\dot{\omega}_2 \\
\dot{\omega}_1
\end{bmatrix} =
\begin{bmatrix}
0 & \dot{\phi}C_{\phi}T_{\theta} & \dot{\theta}S_{\phi}C_{\theta} & -\dot{\phi}S_{\phi}C_{\theta} & \dot{\phi}C_{phi} \\
\dot{x} & 0 & 0 & T & 0 \\
\dot{y} & 0 & 0 & 0 & 1 \\
\dot{z} & 0 & 0 & 0 & 0 \\
\omega_3 & 0 & 0 & 0 & 0 \\
\omega_2 & 0 & 0 & 0 & 0 \\
\omega_1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
C_{\psi}S_{\theta}C_{\phi} + S_{\psi}S_{\phi} \\
S_{\psi}S_{\theta}C_{\phi} - C_{\psi}S_{\phi} \\
C_{\theta}C_{\phi} \\
0 \\
0 \\
0
\end{bmatrix}
+ \nu + W_{\eta}^{-1}\dot{\nu}
\]
Case Study 1- Quadcopter

• Acumen Model
Case Study 2a- Double Pendulum

- Mathematical Model
  - Lagrangian formulation
  - Generalized Coordinates

\[ \forall i \in \{1,...,|q|\}, \quad \frac{d}{dt} \left( \frac{\partial L}{\partial q_i} \right) - \frac{\partial L}{\partial q_i} = Q \]

- description of components

\[ q = (\theta_1, \theta_2) \]

- define total kinetic and potential energy

\[ T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]

\[ V = m_1 g z_1 + m_2 g z_2 \]
Case Study 2a- Double Pendulum

• identify non-conservative forces

\[ Q = 0 \]

• substitution in Euler-Lagrangian equation

\[
\ddot{\theta}_1 = m_2 l_2 (\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)) \\
+ (m_1 + m_2)g \cos(\theta_1))/(-l_1 (m_1 + m_2))
\]

\[
\ddot{\theta}_2 = m_2 l_1 (\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_2)) \\
+ m_2 g \cos(\theta_2))/ -l_1 m_2
\]
Case Study 2- RICEWRIST

- Rehabilitation
- 3-degree of freedom
- Gibral
- Cover different domains
Case Study 2- RICEWRIST

• Mathematical Model
  – Lagrangian formulation
  – Generalized Coordinates

\[ \forall i \in \{1...|q|\}, \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q \]

• description of components

\[ q = (\theta_1, \theta_2, \theta_3) \]

• define total kinetic and potential energy

\[ V = m_1gh_1 + m_2gh_2 + m_3gh_3 \]
\[ = -m_1gl_2 \cos(\theta_1) + m_3gl_3 \sin(\theta_1) \sin(\theta_3) \]
Case Study 2- RICEWRIST

- identify non-conservative forces

\[ Q = 0 \]

- substitution in Euler-Lagrangean equation

\[
\frac{1}{2} \frac{d}{dt} \left( I_2 \frac{\partial \omega_2 \cdot \omega_2}{\partial \theta_2} + I_3 \frac{\partial \omega_3 \cdot \omega_3}{\partial \theta_2} \right) - \frac{1}{2} \left( I_2 \frac{\partial \omega_2 \cdot \omega_2}{\partial \theta_2} + I_3 \frac{\partial \omega_3 \cdot \omega_3}{\partial \theta_2} \right) = 0
\]
Case Study 1- RICEWRIST

• Acumen Model
Summary

• Untyped core formulation.
• Effectively modeling.
• Newton-Euler formulation.
• Lagrangian formulation.
• Partial static derivatives inclusion.
Thank you for your attention!

Questions

Jawad.masood@hh.se

@jamatics